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CONSULTATION OF FUZZY DECISION TABLES TO ALLOW FLEXIBLE DECISION-MAKING

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G. WETS

F. DELEUS

J. VANTHIENEN

Katholieke Universiteit Leuven

Naamsestraat 69, B-3000 Leuven

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G. WETS, F. DELEUS, J. VANTHIENEN
Katholieke Universiteit Leuven
Department of Applied Economic Sciences
Naamsestraat 69, 3000 Leuven, Belgium
E-mail: {Geert.Wets, Filip.Deleus, Jan.Vanthienen}
@econ.kuleuven.ac.be
Tel: +32 16-32.68.80
Fax: +3216-32.67.32

Abstract

In this paper the consultation of fuzzy decision tables (FDTs) is discussed. First, the consultation of classical (crisp) decision tables (DTs) is illustrated. In a crisp DT, the decisions which are made are crisp. This is one of the major strengths of the DT formalism since it allows for easy verification of the represented knowledge. As a result, the DT formalism is helpful to automate the decision-making in complex problem domains. However, this crisp character also prohibits that the imprecision, which is frequently intrinsic to human decision-making, can be expressed. This makes that the decisions made by a DT are sometimes not very intuitive. This imprecision can be captured in a FDT and as such a FDT should be helpful to automate the decision-making in complex real-life environments. We will focus on the decision-making process itself, namely on the derivation of a conclusion given a set of facts. We will see that these facts may be formulated in vague terms or may be a crisp value. We will also discuss in which form a conclusion can be given and we will examine whether these conclusions are more intuitive than in the crisp case.

Index terms

knowledge-based systems, decision tables, fuzzy set theory, consultation, imprecision

1. Introduction

Decision tables (DTs) were originally used as a technique in computer programming [3, 17]. Due to its representational capabilities, the application field has extended later on (up till now) to several other domains with logical complexity. The emphasis has moved towards the power of the decision table to represent complex decision situations in a simple manner, easy to check for anomalies [6, 5]. As such, the DT-formalism can act as a good framework to automate the decision-making process in complex environments.

However, in many situations, the strict classical logic in the DT seems not able to support the decision-making. This is due to the fact that in many real-life applications one is often confronted with ill-structured domain knowledge or imprecise information. This makes it difficult to translate such problems into the classical decision table formalism and as a matter of fact the decisions made are not always what a human decision maker would expect intuitively. As human decision makers we can handle ill-structured problems since we tend to view decision-making problems semantically. This makes that we can reason on a flexible manner with vague and imprecise information. If we want to automate real-life decision problems, we need a way to capture the imprecision into the decision-making framework.

For this reason, fuzzy set theory has been incorporated into the decision table formalism [28]. The use of fuzzy set theory allows decision-making rules to be represented semantically. It allows for reasoning with imprecise, ill-structured knowledge in an exact mathematical way. As such, fuzzy decision tables (FDTs) are aimed to serve as a framework to capture knowledge in a complete and consistent way on one hand and to allow modelling of ill-structured problems and decision-making with vague information on the other. In this way, FDTs are expected to facilitate automatic decision-making in complex real-life problems.

In this paper, we will focus on the decision-making phase itself, which means the reasoning process in a specific situation given that the decision-making environment is already modelled by means of a FDT. First, it will be clarified why crisp decision-making may sometimes give results which are not expected intuitively. Next, it will be investigated how FDTs can give an answer to this problem. We will examine how a FDT can be consulted, namely in which form an input value may be formulated and in which form a conclusion is given. It will also be explained how this conclusion can be derived. The aim of this paper is to examine whether the introduction of fuzziness allows more flexible and human-like decision-making.

This paper is organised as follows. In section 2, we give an overview of some basic concepts of decision tables and fuzzy set theory. In section 3, fuzzy decision tables are presented. In section 4, we will discuss fuzzy decision-making.

2. Basic concepts

In this section, we will provide a brief overview of decision tables and fuzzy set theory. We will define both concepts and explain their major characteristics.

2.1 Decision Tables

A DT is a tabular representation used to describe and analyze procedural decision situations, where the state of a number of conditions jointly determines the execution of a set of actions. Not just any representation, however, but one in which all distinct situations are shown as columns in a table, such that every possible case is included in one and only one column.

2.1.1 Definition

A DT consists of four parts. The *condition subjects* are the criteria which are relevant to the decision making process. They represent the items about which information is needed to take the right decision. Condition subjects are found in the upper left part of the table. The *condition states* are logical expressions determining the relevant sets of values for a given condition. Every condition has its set of condition states. Condition states are found at the right hand side of the table. The *action subjects* describe the results of the decision-making process. They are found in the lower left part of the table. The *action values* are the possible values a given action can take. They are found at the right hand side of the table. These four parts can be defined more formally:

Given :

- $CS = \{CS_i\}$ (i=1..*cnum*) is the set of condition subjects ;
- $CD = \{CD_i\}$ (i=1..*cnum*) is the set of condition domains,

  with CD_i : the domain of condition subject i, i.e. the set of all possible values of condition subject CS_i ;
- $CT = \{CT_i\}$ (i=1..*cnum*) is the set of condition state sets,

  with $CT_i = \{S_{ik}\}$ (k=1..n_i) : an ordered set of n_i condition states S_{ik}.

  Each condition state S_{ik} is a logical expression concerning the elements of CD_i, that determines a subset of CD_i, such that the set of all these subsets constitutes a partition of CD_i (exclusivity criterion)
- $AS = \{AS_j\}$ (j=1..*anum*) is the set of action subjects ;

- $AV =$ $\{AV_j\}$ (j=1..*anum*) is the set of action subject value sets,

with AV_j : the set of all possible values of action subject AS_i .

A decision table DT can then be defined as a function from the Cartesian product of the condition states CT_i to the Cartesian product of the action values AV_j , by which every condition combination is mapped into one (completeness criterion) and only one (exclusivity criterion) action configuration

DT: $CT_1 \times CT_2 \times \ldots \times CT_{cnum} \rightarrow AV_1 \times AV_2 \times \ldots \times AV_{anum}$

In Figure 1 and Figure 2, examples of decision tables are shown. Remark that these two DTs are linked with each other by means of the symbol ^ in the condition subject 'Important customer'. This means that in order to determine whether a customer is important or not, the DT in Figure 2 needs to be consulted.

Main table												
Type of car	A						B					
Important customer ^	Yes			No			Yes			No		
Quantity	<30	30-50	>50	<30	30-50	>50	<30	30-50	>50	<30	30-50	>50
Discount (in %)	10	10	15	5	10	10	5	5	10	0	5	5
Free radio	yes	yes	yes	no	no	yes	no	yes	yes	no	no	no
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Figure 1:Main table

Important customer						
Term of account	< 1 year			≥ 1 year		
Quantity taken during the last year	<200	200-300	>300	<200	200-300	>300
Important customer	no	yes	yes	no	no	yes

Figure 2: Condition subtable

2.1.2 Literature

Work about DTs started in the late fifties with a research project at General Electric called the "Integrated Systems Project". The purpose of this project was to study complex decisions in the context of manufacturing processes. In the early phases of the project, it became clear that current decision techniques, such as flowcharts, were inadequate to model complex processes. Therefore, during the project a new technique was developed to facilitate the representation of complex decision processes. This technique was initially called decision structure tables, but soon the term DTs was introduced. After the pioneer projects the field of DTs grew rapidly. Figure 3 depicts the main points of attention [29].

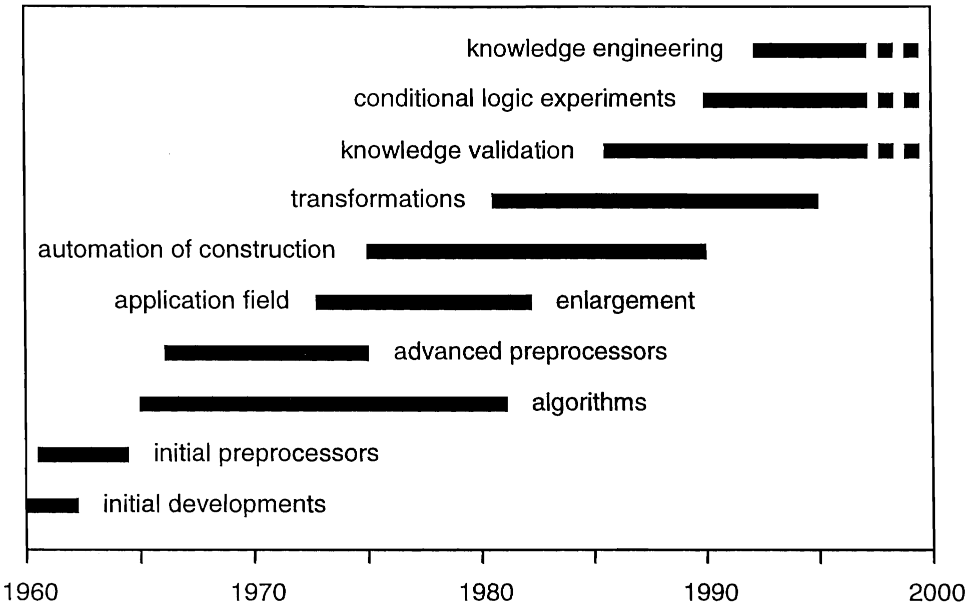

Figure 3: The evolution of the DT technique

In the first period of DT research, much of the work about DTs dealt with the applicability of DTs during the software development life-cycle. Although the way that DTs are used has changed significantly over the years, the advantages of DTs for software engineering purposes are still emphasized by many authors[2, 3, 8, 16, 25, 31].

The next important period in DT research ranges from the mid-'60s to the early-'80s. During this era, generating an optimal execution tree (with respect to minimal execution time or to minimal storage space) from a DT was the main subject of research [13, 14, 17, 19, 21, 30]. In some cases, DTs were designed to be converted to executable program code. This conversion could be done manually or with a pre-processor. Considerable research about DTs went in this direction [4, 33].

During the last two decades, the objective of DT research has moved towards the power of the DT to represent complex decision situations in a simple manner such that they can be checked easily for consistency and completeness. This extension has directed research efforts from the efficient conversion of the table into program code towards the construction process of the table. Furthermore, the application area has been extended from computer programming towards various other domains with logical complexity: information systems analysis, description of systems requirements, management procedures, knowledge validation and engineering, etc. Especially in the field of knowledge-based systems (KBS), the use of DTs has gained some attention. Currently, in the context of KBS, there are three domains where DTs are encountered: mainly during the verification and validation (V&V) process [6, 18, 15], but also as a fast way of executing the expert system [5] and during the knowledge acquisition phase [1 20, 23, 26].

2.1.3 Consultation of decision tables

In the previous section, it was stated that DTs offer a uniform technique for the whole life-cycle of a decision system, going from the knowledge acquisition phase, through the verification and validation process down to the decision-making itself. In this section, it will be clarified how a decision can be made when knowledge is represented in a DT.

When a decision has to be taken, the condition subjects are evaluated, which means that for each condition subject, a condition state is selected. By continuing to choose from the relevant condition states, irrelevant columns are struck out until one specific column is reached. The action configuration of that column will then be executed.

Example

We will illustrate a consultation of the DTs in Figure 1 and Figure 2 given the following facts:

- quantity taken during the last year = 172;
- term of account = 11 months;
- type of car = A;
- quantity = 52;

The consultation of a DT structure always starts with the main table. The first condition subject of this table is evaluated. Because the cars are of type A, the first condition state is selected. Only columns 1 to 6 remain relevant, the other columns have no longer to be considered. Next, one has to determine whether the customer is important or not. Therefore, the condition subtable 'Important customer' is consulted. This gives as result that the customer is not important, being the action configuration of column 1. This knowledge reduces the decision space further to the columns 4-6 of the main table. Finally, the condition subject 'Quantity' is evaluated. As an exact value for this condition subject is available, the set to which this value belongs is selected, which is in this case the third condition state of 'Quantity'. Now a decision can be taken: column 6 is selected, meaning that the customer will receive a discount of 10 % and a free radio.

A DT may be consulted by a human operator or by a program without human intervention. If a DT is consulted in a program, input values are extracted from external databases or from other applications. This is the typical case for control systems. If a DT is consulted by a human decision maker, the decision logic may be consulted visually or the decision maker may be assisted by a consultation manager. This is the case in expert systems. A consultation manager can be built as a question answering system through which the user can give the values for the condition subjects. The questions asked by the system can be in the form of 'What is the quantity ordered?' or 'Is the quantity ordered greater than 50?' or 'Mark one of the following ranges as the relevant state for quantity'. Based on these answers, the DT proposes a decision.

2.1.4 Drawbacks of crisp consultation

We have seen above that based on the input values or the answers of the user, a decision is taken. However, these decisions may sometimes feel unnatural. To clarify this, again the DTs shown in Figure 1 and Figure 2 are used. Consider the situation that a customer has ordered 52 cars of type A and that he is an important customer because the term of his account is 9 months and that he has taken 205 cars during the last year. This would lead to the conclusion that he will receive a 15 %-discount. Suppose that this same customer had ordered 49 cars instead of 52, then he would receive a discount of only 10 %, which is a rather great change in the output, compared to the little change in the input. The same would occur if he had only ordered 198 cars during the last year instead of 205. Then he would be considered as not important and as a result, he would receive only a 10 %-discount. Such jumps in the conclusions are not what a human decision maker expects intuitively.

The cause of this unnatural decision behaviour lies in the fact that we have determined cut-off points above which a customer is seen as being important and below which he is seen as being unimportant. Similarly, we have defined cut-off points at which the customer belongs to another discount category. Such cut-off points are inappropriate here and contradict human intuition. Moreover, as illustrated above, they provide a completely different decision in the event of only minimally changed input data at such a threshold. In the following section, fuzzy set theory is introduced to handle this problem and to allow more gradual and human-like decisions.

2.2 Preliminaries on fuzzy set theory

Fuzzy set theory is based on a recognition that certain sets have imprecise boundaries. Zadeh [34] defines a fuzzy set as "a class of objects with a continuum of grades of membership." More formally, we define a fuzzy set as follows:

Definition

Let U be the universe of discourse. A fuzzy set F on U is characterised by a membership function μ_F: $U \rightarrow [0,1]$, which associates with each element u of U a number $\mu_F(u)$ representing the grade of membership of u in F. $\mu_F(u) = 0$ means non-membership, $\mu_F(u) = 1$ means full membership, and $\mu_F(u)$ with $0 < \mu_F(u) < 1$ means partial membership. Symbolically, $F = \{ \mu_F(u)/u \mid u \in U \text{ and } \mu_F(u) \in [0,1] \}$.

Remark that classical set theory is just a special case of fuzzy set theory in which the membership values can only be 0 or 1.

Fuzzy sets are very popular to define linguistic expressions. Suppose that we have to determine the ages that belong to the category 'old'. If we say that the ages above 70 are 'old' ages, what can we say about age 69, 68, 67,... The question then becomes: can we draw a sharp boundary conceptually in the determination of the 'old' ages. We feel that drawing a cut-off line in this situation is very artificial and not intuitively defensible. This problem can be evaded if we describe the old ages by a fuzzy set. Often, more fuzzy sets are defined on the same universe of discourse, describing different adjectives of the same substantive, then called a linguistic variable. A linguistic variable is a variable the values of which are not numbers but adjectives. In Figure 4, the linguistic variable 'age' is shown. It can be seen that transitions between different values, adjectives, of the linguistic variable are not abrupt, not all-or-nothing but rather gradual. As such, a same person can be middle-aged as well as old to a certain extent.

In a natural language, there are not only adjectives, but also adverbs like 'very', 'more or less',... Such adverbs can be implemented in fuzzy set theory by hedges. A hedge is an operator which changes the membership of the elements of a fuzzy set and as such changes the meaning of that fuzzy set. Examples of linguistic modifiers are the concentration ($\mu_{con(A)}(u)=(\mu_A(u))^2$) and the dilation ($\mu_{dil(A)}(u)=(\mu_A(u))^{1/2}$). The concentration is often used to model the meaning 'very', while the dilation represents often the meaning 'more or less'. In Figure 4, the dotted line represents the adjective 'very old'.

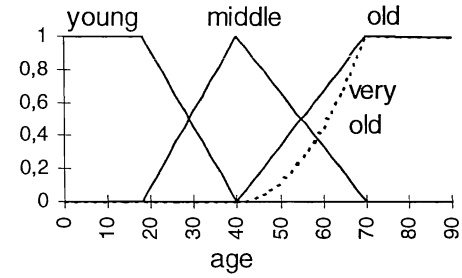

Figure 4: linguistic variable 'age'

However, the membership function is a key concept in fuzzy set theory, the assignment of membership to elements in a fuzzy set is a very difficult problem and is still unsolved at this stage. Dubois and Prade [9] review a number of ideas and methods to study this problem of estimation of membership functions that have been suggested in the literature. A meaningful interpretation of a membership function, which will still be often used later on in this paper, is that of a possibility distribution. It has been introduced by Zadeh (1978) [37]. We will illustrate this interpretation with an example.

Suppose that we don't know the exact age of a person , but we know he is old. Then the membership function of the fuzzy set 'old' gives for each age the possibility that the person has that age. For instance when using Figure 4, if we get the information that someone is old, we know that it is fully possible that he is 75 but that it is only possible to a degree of 0,6 that he is 58 and that it is not possible that he is 32.

3. Fuzzy decision tables

Based on the definitions of DTs given in section 2.1, fuzzy decision tables (FDTs) can be defined. Recall that a DT was defined as a function from the condition part to the action part. The main difference between a crisp DT and a FDT is that in a FDT the condition states and action states can be expressed by fuzzy linguistic terms. Thus, the formal definition given in section 2.1.1 still holds if we allow that to each (condition or action) state a fuzzy set can be assigned [28].

Example

To illustrate the concept of a FDT, the DTs depicted in Figure 1 and Figure 2 will be used. We will fuzzify some of the condition subjects and action subjects to integrate vagueness into the decision model. This is shown in Figure 5 and Figure 6.

Main table												
Type of car	A						B					
Important customer ^	Yes			No			Yes			No		
Quantity	low	medium	high	low	medium	high	low	medium	high	low	medium	high
Discount (in %)	medium	medium	big	small	medium	medium	small	small	medium	no	small	small
Free radio	yes	yes	yes	no	no	yes	no	yes	yes	no	no	no
	1	2	3	4	5	6	7	8	9	10	11	12

Figure 5: Fuzzy decision table

Important customer						
Term of account	low			high		
Quantity taken during the last year	low	medium	high	low	medium	high
Important customer	no	yes	yes	no	no	yes

Figure 6: Fuzzy condition subtable

In this example, the condition subjects 'Important customer', 'Quantity', 'Term of account' and 'Quantity taken during the last year' and the action subjects 'Discount' and 'Important customer' are fuzzy. Concretely, this means that a customer can be partially important and unimportant as the states 'Yes' and 'No' of 'Important customer' are modelled by fuzzy sets. These states are depicted in Figure 7.

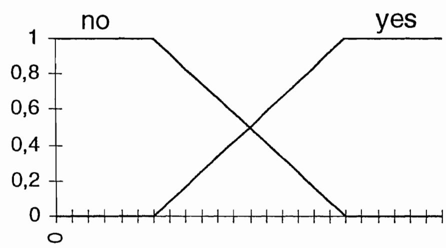

Figure 7: Fuzzy states of important customer

The construction of FDTs can proceed mainly according to the steps of the crisp case, however, some extensions are needed. For example, extra steps are necessary to specify fuzzy sets involved in conditions or actions, some provisions are needed to handle fuzzy decision rules, etc. In the following section, we will investigate whether it is worthwhile to do these extra efforts, namely whether the introduction of fuzziness allows more flexible and human-like decision-making.

4. Fuzzy decision-making

4.1 Overview

In this section, we will discuss how fuzzy decision-making can occur using FDTs. Recall that in a FDT, condition states or action values may be represented by a fuzzy set. Since both conditions and actions can be fuzzified or not four types of FDTs are possible: conditions crisp - actions crisp, conditions fuzzy - actions crisp, conditions crisp - actions fuzzy and conditions fuzzy - actions fuzzy. Of course, some mixed forms are possible but to clarify the influence of fuzzifying conditions and actions we will investigate non-mixed types.

During consultation of a DT, the user, whether it is a human operator or a program, provides values for the condition subjects. In the case of crisp consultation of a crisp DT as shown above, the user has to give an input value which belongs to exactly one of the sets defining the condition states of the condition subject at issue or he has to select one of the condition states tabulated in the table. If fuzziness is allowed, one has more flexibility. In general, the input value for a condition subject is then a fuzzy set. In many cases, this input fuzzy set will contain only one element with membership value one, which means that the input is simply a numerical value. We call this crisp consultation. If an exact numerical value for a condition subject is not known or cannot be determined, the input may be a fuzzy set, eventually representing a linguistic expression.

This gives raise to the following 8 possible combinations:

	CONDITIONS	ACTIONS	INPUT
1	Crisp	Crisp	Crisp
2	Crisp	Crisp	Fuzzy
3	Fuzzy	Crisp	Crisp
4	Fuzzy	Crisp	Fuzzy
5	Crisp	Fuzzy	Crisp
6	Crisp	Fuzzy	Fuzzy
7	Fuzzy	Fuzzy	Crisp
8	Fuzzy	Fuzzy	Fuzzy

Table 1: Possible types of consultation

It is important to note that either fuzzy set on the condition domain can be a valid input. This means that linguistic expressions, other than these in the condition states, may be used as input for a consultation. For instance, let a condition subject 'age' have two states, 'young' and 'old', each defined by a fuzzy set. Valid input values may be 'very old', 'not so young', 'about 20', 'exactly 36',... Remark that, since a crisp set is just a special case of a fuzzy set, fuzzy consultation may also be performed on a crisp DT.

4.2 Decision-making strategies

4.2.1 General outline

In the case of crisp consultation of a crisp DT, we have seen that decision-making in fact boils down to merely checking with each column of the table to match perfectly a given combination of condition values. However, if fuzziness is allowed, more columns can give a partial matching as it is common that neighbouring states overlap to a certain extent. Also, if the condition states do not overlap, for instance in the case of a crisp DT, more columns can give a partial matching if the input value is a genuine fuzzy set. The question then becomes how an appropriate decision can be taken. At this point, it is useful to distinguish between crisp, each other excluding actions on one hand and fuzzy actions on the other.

- If an action subject is crisp, one has to choose one action value of that action subject. This reasoning will be based on similarity measures.
- If an action subject is fuzzy, a more complex reasoning method, based on the compositional rule of inference may be performed. This method proceeds as follows. First, in each column the action value of the action subject at issue is changed based on the input values. Next, these individually changed action values are combined, so taking into account the knowledge of the whole DT. By aggregating the individual action values, one new fuzzy set is calculated for the fuzzy action subject. This new fuzzy set can then be interpreted by a linguistic expression or a numerical value may be chosen. Consequently, as opposed to the case of crisp actions, if an action subject is fuzzy, the decision made can be an action value which is not tabulated in the DT. The reasoning process itself generates a new decision depending on the given input values.

In the following sections, we will explain in more detail how the reasoning may occur in the cases depicted in Table 1.

4.2.2 FDTs without fuzzy actions

4.2.2.1 Crisp conditions - Crisp input (case 1)

We start with the case that both the condition states and the action values are represented by crisp sets. Remark that this is the case of a classical DT. To illustrate the decision-making, we will use the DT depicted in Figure 8.

demand	≤ 20		> 20	
supply	≤ 20	> 20	≤ 20	> 20
price	level 2	level 1	level 3	level 2

Figure 8: crisp DT of the decision problem

Suppose that we know that demand will be 14 and supply 24, a consultation of the DT then gives as conclusion that the price will be on level 1, being the action configuration of the second column. If the demand is higher, say 18, then the price stays at the same level. However, if the price is still a little bit higher, for instance 22 and supply a little bit lower, for instance 19, then column 3 is selected. This implies that the price level jumps from the lowest one to the highest one as a result of a rather small change in the input values. As stated above, the cause of this unnatural behaviour is that we have determined cut-off points to define the different condition states. However, a real economical system cannot be modelled by determining such cut-off points. For this reason, we will fuzzify the condition states.

4.2.2.2 Fuzzy conditions - Crisp input (case 3)

Now the crisp states are replaced with fuzzy sets, respectively representing the meanings ‘low’ and ‘high’. As such, the decision situations can be modelled in terms of linguistic expressions, as shown in Figure 9. The definitions of the condition states are depicted in Figure 10.

demand	low		high	
supply	low	high	low	high
price	level 2	level 1	level 3	level 2

Figure 9: FDT with crisp actions

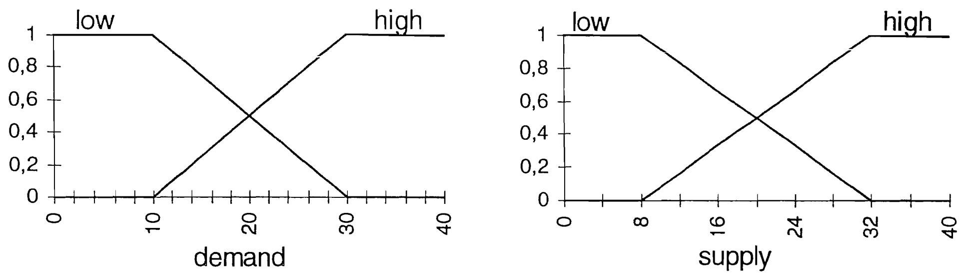

Figure 10: definition of the condition states

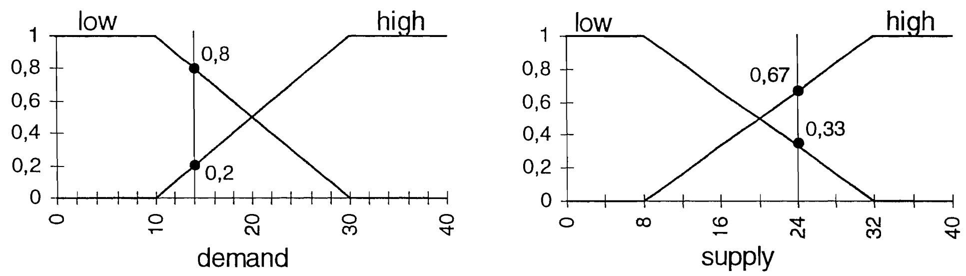

Figure 11: crisp consultation of fuzzy condition states

Now, consider again the situation that demand is 14 and supply 24. This situation is illustrated in Figure 11. These input values are not really low and not really high and as a matter of fact, more columns of the FDT will give a partial matching. The essence of the reasoning process now consists of calculating the different degrees of matching between the specific input situation and the respective condition parts of the columns. Therefore, first the degrees of membership of the input values in the different fuzzy sets are determined, which is shown in Figure 11. Next, these membership values are combined by means of a t-norm. With *min* as t-norm, one obtains the following results.

Column 1 : min (0,8 ; 0,33) = 0,33

Column 2 : min (0,8 ; 0,67) = 0,67

Column 3 : min (0,2 ; 0,33) = 0,22

Column 4 : min (0,2 ; 0,67) = 0,22

Column 2 has the highest degree of matching, which means that the conclusion will be that the price will be on level 1. However, the results indicate that also level 2 and to a lesser degree also level 3 are possible.

In the crisp case, we considered the input situation that demand equals 22 and supply equals 19. The results of consultation of the FDT with these input values are the following:

Column 1 : min (0,4 ; 0,54) = 0,4

Column 2 : min (0,4 ; 0,46) = 0,4

Column 3 : min (0,6 ; 0,54) = 0,54

Column 4 : min (0,6 ; 0,46) = 0,46

Like in the crisp case, level 3 is now the most possible level. However, this consultation gives us more information than simply saying that the price jumps from level 1 to level 3. Now we see that the difference with the other columns is very small, which can be useful information for a human decision maker.

4.2.2.3 Fuzzy conditions - Fuzzy input (case 4)

Until now, the inputs were always numerical values. However, in many real-life situations a crisp numerical value is not known. For instance, in our example, if the DT is used in a micro-economical model, it is possible that the enterprise knows her supply curve so that the input value for 'supply' can be a numerical value. If the enterprise has a limited number of well-known customers, it may be possible that also the demand curve is known so that for 'demand' also the input can be numerical. However, in most cases, the demand curve is not known and must be forecasted. In that case, a fuzzy input is more appropriate for 'demand'. The more, if the DT is used in macro-economical models, in most cases a exact numerical value will not be known neither for demand nor for supply. For instance, if the GNP of a country is needed in a model, a fuzzy set, eventually representing a linguistic expression can be more appropriate as input than an exact value.

Based on the input fuzzy sets, now a decision has to be taken. Therefore, the degree of matching between the given combination of condition values and each column should be evaluated. A given combination of condition values can be represented by a multi-dimensional fuzzy set by performing a t-norm on the respective fuzzy sets. In the same way, the condition part of each column may also be considered as a multi-dimensional fuzzy set. The degree of matching between a combination of input values and each column can then be calculated by using a similarity measure. Several *SM*s have been proposed in the literature [38]. A *SM* which in most cases shows good performance is the following:

$$SM(A', A) = \sup_i T\left[\mu_{A'}(x_i), \mu_A(x_i)\right]$$

In most cases, the similarity between two multi-dimensional fuzzy sets needs to be computed. In general, this calculation is very complicated since it involves for a n-antecedent system a n-dimensional matrix operation. Because of these complex calculations, Turksen & Tian (1995) [24] have proposed a simplification. They prove that, if the same t-norm is used to calculate the SM and to the connective AND then, like in the case of numerical input values, the similarity between the combination of input values and the condition part of a column may be computed by performing a t-norm on the similarity values between the respective input values and the condition states.

For example, consider the situation where demand is *more-or-less high* and supply is *not low but less than medium.* One possible representation of this input is depicted by the dotted lines in Figure 12.

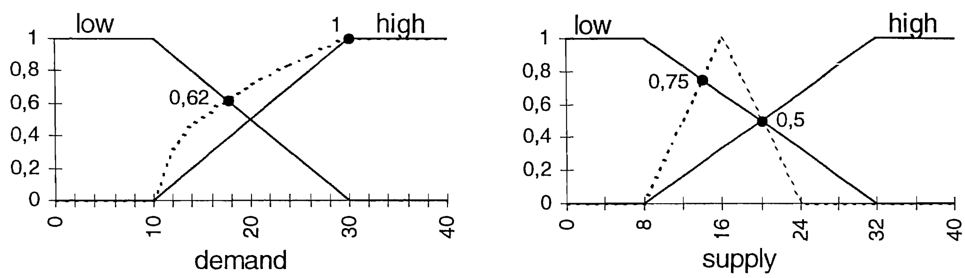

Figure 12: fuzzy consultation of fuzzy condition states

First, for each condition subject the similarity between the input value and each condition state is evaluated, as shown in Figure 12. For instance, the similarity between the input value for demand and a low demand is 0,62. Next, these degrees of similarity are combined by the min-operator to obtain the matching of the input with each column. This gives the following results:

Column 1 : min (0,62 ; 0,75) = 0,62

Column 2 : min (0,62 ; 0,5) = 0,5

Column 3 : min (1 ; 0,75) = 0,75

Column 4 : min (1 ; 0,5) = 0,5

We see that column 3 has the highest degree of matching, such that the conclusion will be that the price will be on level 3, the highest level. At the same time, we see that also column 1 gives a rather high degree of matching, meaning that also level 2 has a rather high possibility of being the future price level.

4.2.2.4 Crisp conditions - Fuzzy input (case 2)

As stated above, a crisp set is just a special case of a fuzzy set. This makes that fuzzy consultation can also be performed on a crisp DT. This is of great value because existing (crisp) DTs can then be utilised. For example, consider the crisp DT in Figure 8. When consulting these DT given the facts that demand is *more-or-less high* and supply is *not low but less than medium*, one obtains the graphical representation as illustrated in Figure 13.

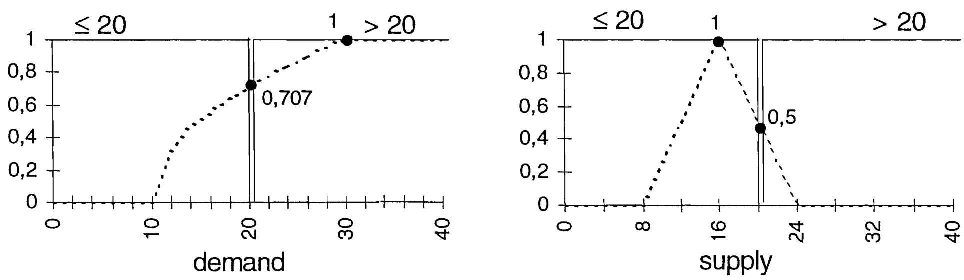

Figure 13: fuzzy consultation of crisp condition states

Again the similarity between the input values and the condition states are calculated and combined by means of a t-norm. This gives the following results.

Column 1 : min (0,707 ; 1) = 0,707
Column 2 : min (0,707 ; 0,5) = 0,5
Column 3 : min (1 ; 1) = 1
Column 4 : min (1 ; 0,5) = 0,5

Column 3 has the highest degree of matching, but again the results indicate the possibilities of other levels.

4.2.3 FDTs with fuzzy actions

Until now, in all the examples, the consultation of the FDT proposes a decision and at the same time indicates that still other alternatives are possible. However, since the action values are represented by crisp states, the knowledge that still other alternatives are possible cannot automatically be integrated in the final conclusion itself. Therefore, it is worthwhile to fuzzify the actions. In the example, we replace the crisp action values 'level 1', 'level 2' and 'level 3' with fuzzy sets denoting the linguistic terms 'low', 'medium' and 'high'. Figure 15 shows these values of the linguistic variable 'price'. The decision problem can now be modelled in a very intuitive manner, as illustrated in Figure 14.

demand	low		high	
supply	low	high	low	high
price	medium	low	high	medium

Figure 14: FDT with fuzzy actions

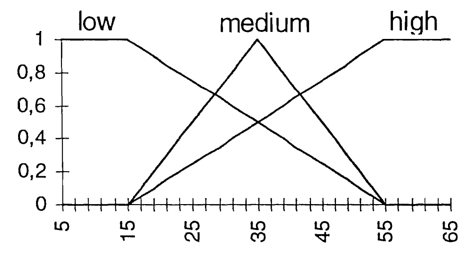

Figure 15: definition of the action values

The decision-making process consists in calculating the output fuzzy set for that action subject. First, the action value of each individual column is changed depending on the inputs. Therefore, we will use the generalised modus ponens, which is defined as:

if X is A then Y is B

X is A'

Y is B'

where A' represents the data and B' the inferred result. A and A' are defined on X and B and B' on Y.

To infer the result B', one usually assumes that the fuzzy rule "if x is A then y is B" can be represented as a relation R. This R is a multi-dimensional fuzzy set with universe of discourse the cartesian product of the universes of discourse of A and B. B' can be inferred through the composition, denoted as $\circ$, of A' and R. This type of inference was proposed by Zadeh (1973) [35] and is called the compositional rule of inference. More formally, $B' = A' \circ R$. To implement the compositional rule of inference Zadeh (1975) [36] proposes the following. Because B' is defined on Y, A' on X and R on $X \times Y$, $B' = \text{proj}(A' \circ R)$ on Y. This is equal to $\sup_x T(\mu_A(x), \mu_R(x, y))$. Usually in this expression *max* and *min* are used to compute the supremum and the t-norm respectively.

In the context of a FDT, "X is A" and "Y is B" are expressed in the condition part and action part respectively, "X is A' " is a given combination of condition values, and "Y is B' " is an action to take. Note here that A' and B' are generally different from A and B. As a matter of fact, "Y is B' " is a new piece of information, knowledge or action that is derived from the FDT.

An important issue is the selection of an appropriate implication function I to define the fuzzy relation R. Dubois & Prade (1996) [10] state that this selection needs to be based on the semantics underlying the fuzzy rule. They distinguish two interpretations of fuzzy rules which serve different semantics: conjunction-based and implication-based. The first is widely adopted in fuzzy control, while the second is often used in approximate reasoning. In the context of the consultation of FDTs, we are dealing with decision problems which can be solved using some predefined knowledge. Clearly this knowledge is expressed in terms of rules which express some genuine implication. Therefore, we will only deal with fuzzy rules using the implication-based model. In the implication based model, a rule is seen as a piece of knowledge which puts a restriction on possible values of the action subject. Two types of implication-based rules can be distinguished: certainty rules and gradual rules. Certainty rules are of the form 'the more x is A, the more certain y lies in B'. For instance 'the younger a person is, the more certain the person is single'. Gradual rules correspond to statements of the form 'the more x is A, the more y is B'. An example of a gradual rule is 'the redder the tomato, the riper it is'. Dubois and Prade show that in the case of certainty rules the Kleene-Dienes implication function ($a \rightarrow b = \max(b, 1-a)$) needs to be used, while in the case of gradual rules, the Gödel-implication ($a \rightarrow b = 1$ for $a \leq b$; $a \rightarrow b = b$ for $a > b$) is the most appropriate one.

4.2.3.1 Fuzzy conditions - Crisp input (case 7)

Now consider again the decision situation of Figure 14. We will use the same input values as those used in the case of crisp action values. In the first consultation, the input value for demand was 14 and for supply 24 as illustrated in Figure 11. We saw that column 2 had the highest matching. Therefore, we first calculate the new action value for the second column. Because the rules underlying the columns are of the gradual type, for instance 'the more demand is low and the more supply is high, the more price is low', we use the Gödel implication function. The result is shown in Figure 16 in which the dotted line represents the new action value of the second column and the straight line the original value 'low'.

Figure 16: modified action value of column 2

In this figure, we see that because the demand is not really low and the supply is not really high, higher values of price get a higher membership value, which means a higher possibility. We can also interpret this as follows. Since the original rule cannot be fired completely, the restriction he lies on the possible values of the action subject has to be less strict, which means that more values get a higher possibility. This same procedure needs to be performed on all the columns with a positive degree of matching.

Next, all the individually changed action values have to be combined into one resulting action value. As stated above, in the implication-based model, a rule puts a constraint on the set of possible values. If more than one rule can be fired partially, each of them generates a certain constraint. It is intuitive that these constraints have to be combined by means of an AND-operator. When using the min-operator for combining the different action values of the action subject price, one obtains the final action value illustrated in Figure 17.

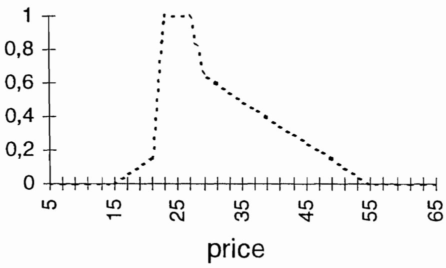

Figure 17: new action value

In this new action value, we see the influence of the other columns. While in the case of crisp actions the matching factors only indicated that also other columns should be taken into account, in the case of fuzzy actions this knowledge is integrated in the final action value. If necessary, this new action value can be interpreted by a linguistic expression or a numerical value can be chosen. As illustration, Figure 18 shows the new action values for the situations in which demand is 18 and supply is 24 and in which demand is 22 and supply 19 respectively. Then we see that the new fuzzy set representing the conclusion gradually moves to higher values of prices, which means that the higher values become more possible.

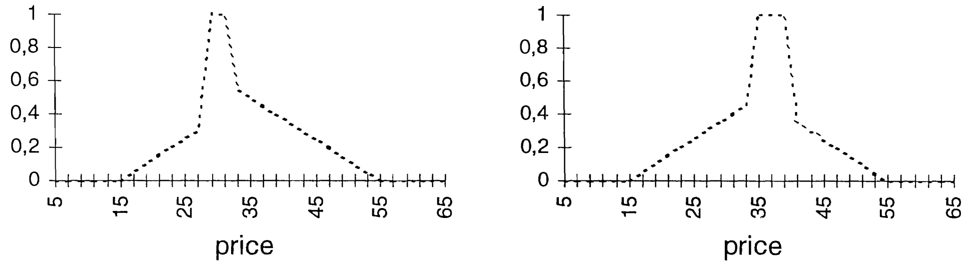

Figure 18: new action values

4.2.3.2 Fuzzy conditions - Fuzzy input (case 8)

Exactly the same procedure can also be used when the input values are fuzzy sets. Consider again the input situation in which demand is ***more-or-less high*** and supply is ***not low but less than medium.*** This situation was represented in Figure 12. The output value for this consultation is represented in Figure 19.

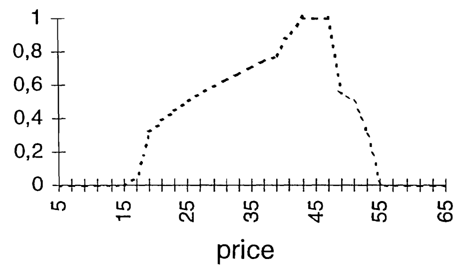

Figure 19: new action value

Again, we see the influence of the different columns. The new action value represents a price between high and medium.

4.2.3.3 Crisp conditions - Fuzzy or crisp input (cases 5 and 6)

Since a crisp set is a special case of a fuzzy set, these two situations are special cases of the general case in 4.2.3.2.

4.3 Fuzzy consultation manager

We stated above that a classical DT can be consulted visually or by means of a consultation manager. If fuzziness is allowed, visual consultation is rather difficult as the FDT itself does not make clear which new action value comes about. A key concept in fuzzy set applications is the membership function and this is not explicitly shown in the FDT formalism. For this reason, a good consultation manager is very important. In such a consultation manager, the user needs to have the choice in which form he gives the input values. He can give a numerical value or a fuzzy set. The consultation manager can also contain a database of definitions of linguistic expressions so that the user can give a linguistic expression as an input value. For the output, in the case of crisp actions, the system can give the degrees of matching with the respective action values. In the case of fuzzy actions, the system can simply give the new generated fuzzy set, leaving the interpretation over to the user. The consultation manager can also give a numerical value or he can give an answer in the form of a linguistic expression.

5. Conclusions

DTs are useful to represent complex decision situations in a simple fashion, easy to check for anomalies. In many real life applications, the use of crisp DTs may sometimes show difficulties, as often some imprecision is involved. Disregarding this imprecision may lead to conclusions which are not very intuitive. In such cases, a FDT may be more appropriate since the domain knowledge may be modelled in vague or linguistic terms. It may also happen that at the moment of the decision-making itself, an exact value for a decision variable is not known. The input value may then be a linguistic term or in general a fuzzy set. We have shown that such a fuzzy consultation may be performed on fuzzy DTs as well as on crisp DTs. Depending on the problem domain, the decisions made can be crisp or fuzzy. If the actions are crisp, one action value is selected, but the consultation also indicates possibility degrees for the other actions. If an action is fuzzy, the conclusion is in general a new derived fuzzy set. This new fuzzy set incorporates the vagueness of the problem domain and the input together with the knowledge of the whole decision table. This allows a flexible, gradual and human-like decision behaviour within the decision table formalism.

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